Ambiguities of theoretical parameters and CP/T violation in neutrino factories

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Abstract

We study the optimal setup for observation of the CP asymmetry in neutrino factory experiments — the baseline length, the muon energy and the analysis method. First, we point out that the statistical quantity which has been used in previous works doesn't represent the CP asymmetry. Then we propose the more suitable quantity, $\equiv \chi_2^2$, which is sensitive to the CP asymmetry. We investigate the behavior of χ_2^2 with ambiguities of the theoretical parameters. The fake CP asymmetry due to the matter effect increases with the baseline length and hence the error in the estimation of the fake CP asymmetry grows with the baseline length due to the ambiguities of the theoretical parameters. Namely, we lose the sensitivity to the genuine CP-violation effect in longer baseline.

1 Introduction

The observation of the atmospheric neutrino anomaly by Super-Kamiokande [1] provided us with convincing evidence that neutrinos have non-vanishing masses. There is another indication of neutrino masses and mixings by the solar neutrino deficit [2, 3, 4, 5, 6].

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Assuming three generations of the leptons, we denote the lepton mixing matrix, which relates the flavor eigenstates ($\alpha = \mathrm{e}, \mu, \tau$) with the mass eigenstates with mass $m_i (i = 1, 2, 3)$, by

$$U_{\alpha i} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13}\mathrm{e}^{\mathrm{i}\delta} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}\mathrm{e}^{\mathrm{i}\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13}\mathrm{e}^{\mathrm{i}\delta} \end{pmatrix}_{\alpha i}, \quad (1)$$

where $c_{ij}(s_{ij})$ is the abbreviation of $\cos\theta_{ij}(\sin\theta_{ij})$. Then the atmospheric neutrino anomaly gives an allowed region for $\sin\theta_{23}$ and the larger mass square difference ($\equiv \delta m^2_{31}$). The solar neutrino deficit provides allowed regions for $\sin\theta_{12}(\equiv \delta m^2_{21})$.

On the other hand, there is only an excluded region for $\sin\theta_{13}$ from reactor experiments [8]. Furthermore there is no constraint on the CP violating phase δ. The idea of neutrino factories with muon storage rings were proposed [9] to determine these mixing parameters, and attracted the interest of many physicists [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

However we have some questions concerning the previous analyses of the CP-violation effect. In many analysis, the muon energy of E_μ is assumed to be rather high. This seems strange since CP/T violation arises as a three generation effect [21, 19, 22, 23]. Indeed we can derive very naively that $E_\mu \sim 30$GeV, lower by factor 2, is the most efficient for $L = 3000$ km [19] while $E_\mu \sim 50$GeV is often assumed. Furthermore the fake CP-violation effect due to the matter effect[24] increases with baseline length. The ambiguity in the estimate for the fake CP violation increases with baseline length. Taking into account this ambiguity in the analysis, the sensitivity to CP violation will be decreased as baseline length increases. It is unlikely that we can observe the CP-violation effect with such a long baseline. We discuss these problems[23].

2 Statistical quantity

As an experimental setup, we consider that N_μ muons decay at a muon ring. The neutrinos extracted from the ring are detected at a detector if E_ν is larger than a threshold energy E_{th}. The detector has mass M_{detector} and contains N_{target} target atoms. We assume that the neutrino-nucleon cross section σ is proportional to neutrino energy as

$$\sigma = \sigma_0 E_\nu, \quad (2)$$

The expected number of appearance events in the energy bin $E_{j-1} < E_\nu < E_j$ ($j = 1, 2, \ldots, n$) is then given by

$$N_j(\nu_\alpha \to \nu_\beta; \delta) \equiv \frac{N_\mu N_{\mathrm{target}}\sigma_0}{\pi m_\mu^2}\frac{E_\mu^2}{L^2}\int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu) P(\nu_\alpha \to \nu_\beta; \delta)\frac{\mathrm{d}E_\nu}{E_\mu}, \quad (3)$$

where m_μ is the muon mass.

To estimate the sensitivity for the CP-violation effect the following statistics is usually used:

$$\chi_1^2(\delta_0) \quad \equiv \quad \sum_{j=1}^{n} \frac{[N_j(\delta) - N_j(\delta_0)]^2}{N_j(\delta)} + \sum_{j=1}^{n} \frac{[\bar{N}_j(\delta) - \bar{N}_j(\delta_0)]^2}{\bar{N}_j(\delta)} \tag{4}$$

n is the number of bins. Since the CP violation is absent if $\sin\delta = 0$, namely $\delta = 0$ or $\delta = \pi$, we need to check that $N_j(\delta)$ is different from $N_j(\delta_0)$ with $\delta_0 \in \{0, \pi\}$ to insist that CP violation is present.

We can claim that $N_j(\delta)$ is different from $N_j(\delta_0)$ at 90% confidence level, if

$$\chi_1^2 \quad \equiv \quad \min(\chi_1^2(0), \chi_1^2(\pi)) > \chi^2_{90\%}(n) \tag{5}$$

holds. Here $\chi^2_{90\%}(n)$ is the χ^2 value with n degrees of freedom at 90% confidence level.

To see the behavior of χ_1^2, we make use of high energy approximation which is valid for $E_\nu \gtrsim (\delta m_{31}^2 L)/4$:

$$\chi_1^2(\delta_0) \propto E_\mu \frac{J_{/\delta}^2}{A} \left\{ (\cos\delta \mp 1) \left[1 - \frac{1}{3} \left(\frac{a(L)L}{4E_\nu^{\rm peak}} \right)^2 \right] \right\}^2 \tag{6}$$

Here $E_\nu^{\rm peak}$ is the neutrino energy which gives the maximum valu of the initial neutrino flux f_{ν_α} and $a(L)$ is the effective mass square due to matter effect calculated using Preliminary Reference Earth Model(PREM)[27, 28]. We find that χ_1^2 is an increasing function of E_μ. Thus we can obtain arbitrary large χ_1^2, and we can seemingly achieve arbitrary high sensitivity to search for the CP-violation effect, by increasing muon energy. Thus the higher energy appears to be preferable to observe the CP-violation effect as long as we employ χ_1^2. It is important, however, to note that χ_1^2 has nothing to do with the imaginary part of the mixing matrix in high energy limit. The CP violation is brought about by the only imaginary part of the mixing matrix, which is proportional to $\sin\delta$ in our parameterization. χ_1^2 is relevant with CP violation through unitarity[22].

Therefore we need to consider a statistical quantity which is sensitive to the imaginary part of the lepton mixings. As such a statistics we consider the following quantity:[14, 15]

$$\chi_2^2(\delta_0) \equiv \sum_{j=1}^{n} \frac{[\Delta N_j(\delta) - \Delta N_j(\delta_0)]^2}{N_j(\delta) + \bar{N}_j(\delta)} \tag{7}$$

Here $\Delta N_j(\delta) \equiv N_j(\delta) - \bar{N}_j(\delta)$. It is required

$$\chi_2^2 \quad \equiv \quad \min(\chi_2^2(0), \chi_2^2(\pi)) > \chi^2_{90\%}(n) \tag{8}$$

to claim that CP violation effect is observed.

In the high energy limit

$$\chi_2^2(\delta_0) \propto \frac{L^2}{E_\mu} \frac{J_{/\delta}^2}{A} \left\{ \sin\delta + \frac{1}{3} \frac{a(L)L}{4E_\nu^{\rm peak}} (2\cos 2\theta_{13} - 1)(\cos\delta \mp 1) \right\}^2 \tag{9}$$

($-$ for $\delta_0 = 0$ and $+$ for $\delta_0 = \pi$), where

$$J_{/\delta} \equiv \frac{\delta m^2_{21}}{\delta m^2_{31}} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13}, \tag{10}$$

$$A \equiv \sin^2\theta_{23} \sin^2 2\theta_{13}. \tag{11}$$

To see CP-violation effect is to measure $J_{/\delta} \sin\delta$[25, 26]. In this respect χ^2_2 gives a good standard to observe CP violation.

3 Feasibility of CP violation search in presence of the ambiguities of the parameters

In this section we study the asymmetry with χ^2_2. The values of all theoretical parameters will have ambiguities in practice, and hence we cannot estimate $\Delta N_j(\delta_0)$ precisely. The genuine CP-violation effect will be absorbed into the ambiguity of $\Delta N(\delta_0)$ if the ambiguity of $\Delta N(\delta_0)$ is large. Therefore we must examine whether the CP-violation effect can be absorbed in the ambiguities of the parameters.

Suppose that we use the parameters $\tilde{x}_i \equiv \{\tilde{\theta}_{kl}, \delta\tilde{m}^2_{kl}, \tilde{a}(L)\}$, which are different from the true values $x_i \equiv \{\theta_{kl}, \delta m^2_{kl}, a(L)\}$, to calculate $N_j(\delta_0)$ and $\bar{N}_j(\delta_0)$. We will estimate the fake CP violation due to the matter effect as

$$\Delta\tilde{N}_j(\delta_0) = \tilde{N}_j(\delta_0) - \tilde{\bar{N}}_j(\delta_0), \tag{12}$$

are evaluated from eqs.(3). We then obtain

$$\tilde{\chi}^2_2(\delta_0) \equiv \sum_{j=1}^{n} \frac{[\Delta N_j(\delta) - \Delta\tilde{N}_j(\delta_0)]^2}{N_j(\delta) + \bar{N}_j(\delta)} \tag{13}$$

instead of $\chi^2_2(\delta_0)$. The observed asymmetry $\Delta N_j(\delta)$ consists of the genuine CP-violation effect and the fake one due to the matter effect. We have to subtract the matter effect, but we cannot estimate precisely the fake CP violation $\Delta\tilde{N}_j(\delta_0)$ due to the ambiguities of the parameters. In such a case the sensitivity to CP-violation search gets worse once the ambiguities of the parameters are taken into account, since it is always possible to take $\Delta\tilde{N}_j(\delta_0)$ to satisfy

$$\left|\Delta N_j(\delta) - \Delta\tilde{N}_j(\delta_0)\right| \leq \left|\Delta N_j(\delta) - \Delta N_j(\delta_0)\right|, \tag{14}$$

or equivalently

$$\tilde{\chi}^2_2 \leq \chi^2_2, \tag{15}$$

by adjusting $\tilde{x}_i$'s. We can further argue that we lose more sensitivity as the baseline length gets longer. Let us illustrate the outline described above in detail. The CP asymmetry of probabilities

$$A(\{x_i\},\delta) \equiv P(\nu_\alpha \to \nu_\beta; \{x_i\},\delta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; \{x_i\},\delta) \tag{16}$$

consists of the genuine CP asymmetry $A_{\rm CPV}(\{x_i\},\delta)$ and the fake one $A_{\rm CPM}(\{x_i\},\delta)$, so that

$$A(\{x_i\},\delta) = A_{\rm CPV}(\{x_i\},\delta) + A_{\rm CPM}(\{x_i\},\delta). \tag{17}$$

We need to subtract $A(\{x_i\},\delta_0)$ from $A(\{x_i\},\delta)$, but instead we subtract $A(\{\tilde{x}_i\},\delta_0)$ due to the ambiguities of the parameters and obtain

$$\tilde{A}_{\rm CPV}(\delta) \equiv A_{\rm CPV}(\{x_i\},\delta) + A_{\rm CPM}(\{x_i\},\delta) - A(\{\tilde{x}_i\},\delta_0). \tag{18}$$

Here $A_{\rm CPV}$ and $A_{\rm CPM}$ can be estimated using high energy approximation as

$$\begin{aligned} A_{\rm CPM}(\{x_i\},\delta) &\simeq \frac{1}{3}\left[2\sin^2\theta_{23}\sin^2 2\theta_{13}\cos 2\theta_{13}\right. \\ &+ \left.(2\cos 2\theta_{13}-1)J_{/\delta}\cos\delta\right]\frac{a(L)L}{4E_\nu}\left(\frac{\delta m^2_{31}L}{4E_\nu}\right)^3 \qquad (19) \\ A_{\rm CPV}(\{x_i\},\delta) &= \left(\frac{\delta m^2_{31}L}{4E_\nu}\right)^3 J_{/\delta}\sin\delta. \qquad (20) \end{aligned}$$

The factor

$$\frac{2}{3}\sin^2\theta_{23}\sin^2 2\theta_{13}\cos 2\theta_{13}\frac{a(L)L}{4E_\nu} \tag{21}$$

in eq.(19) is expected to be much larger than $J_{/\delta}$ in eq.(20) with a long baseline . Thus the ambiguity of the fake CP-violation effect, $A_{\rm CPM}(\{x_i\},\delta) - A(\{\tilde{x}_i\},\delta_0)$, can absorb the genuine CP-violation effect $A_{\rm CPV}$, so that $\tilde{A}_{\rm CPV}$, or equivalently $\tilde{\chi}^2_2$, becomes significantly small. The condition to observe CP-violation effect in 90% confidence level, say, is again given by

$$X^2_2 \equiv \min_{\{\tilde{x}_i\}} \tilde{\chi}^2_2 > \chi^2_{90\%}(n). \tag{22}$$

We present in Figs.1 the required value of $N_\mu M_{\rm detector}$ obtained from eq.(22) to observe the CP-violation effect in 90% confidence level. All the parameters are assumed to have ambiguities of 10 %. We find that we cannot observe the genuine CP-violation effect when L is larger than 1000 km. We can qualitatively understand it by eqs.(19) and (20). It is seen that

$$\frac{A_{\rm CPV}}{A_{\rm CPM}} = 3\frac{J_{/\delta}\sin\delta}{2\sin^2\theta_{23}\sin^2 2\theta_{13}\cos 2\theta_{13} + (2\cos 2\theta_{13}-1)J_{/\delta}\cos\delta}\frac{4E}{a(L)L} \tag{23}$$

is a decreasing function of L, which means that the sensitivity to the CP violation is lost as the baseline length gets larger. The condition on L is roughly estimated by $A_{\rm CPV}/A_{\rm CPM} \gtrsim 1$, or

$$L \lesssim \frac{4E}{a(L)}\frac{3J_{/\delta}\sin\delta}{2\sin^2\theta_{23}\sin^2 2\theta_{13}\cos 2\theta_{13} + (2\cos 2\theta_{13}-1)J_{/\delta}\cos\delta}. \tag{24}$$

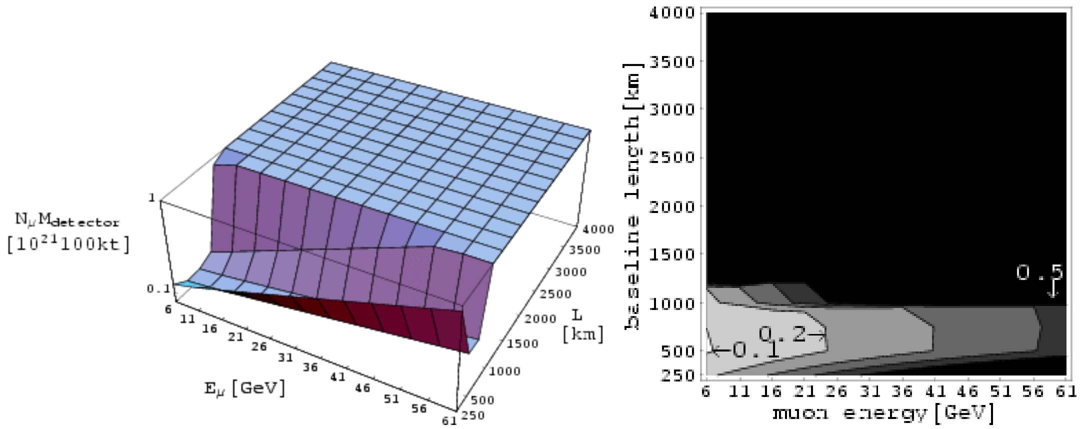

Figure 1: Necessary value of $N_\mu M_{\rm detector}$ to observe the CP-violation effect as a function of muon energy and baseline length, for $\delta = \pi/2$ and $E_{\rm th} = 1$GeV. Here ($\sin\theta_{13}$,$\sin\theta_{23}$,$\sin\theta_{12}$, δm^2_{31},δm^2_{21}) = (0.1, $1/\sqrt{2}$, 0.5, 3×10^{-3}eV^2, 10^{-4}eV^2) and $a(L)$ is calculated using PREM. The ambiguities of the theoretical parameters are assumed to be 10 %. Hence these graphs are obtained using not χ^2_2 but X^2_2. The sensitivity to the genuine CP asymmetry is lost in long baseline region such as $L \gtrsim 1250$km as we estimate in eq.(25).

For the parameters used in Fig.1,

$$L \lesssim 1250\text{km}. \tag{25}$$

4 Summary and Discussion

We discussed the optimum experimental setup and the optimum analysis to see the CP violation effect.

We examined how to analyze the data of experiments to confirm the naive estimation. We studied with two statistical quantities, χ^2_1 (eq.(5)) and χ^2_2 (eq.(8)). Usually χ^2_1 is used in analyses of neutrino factories. We can test by this whether the data can be explained by the hypothetical data calculated assuming no CP-violation effect. We saw, however, that this quantity is sensitive has information for mainly the CP conserved part of the oscillation probability in high energy region. Hence we concluded that it is difficult to measure the CP violation by using this quantity. On the other hand, we can test with χ^2_2 whether the asymmetry of oscillation probabilities of neutrinos and antineutrinos exists. We have seen that χ^2_2 is sensitive to the CP violating part of the oscillation probability, and thus it is suitable quantity to measure the CP violation.

Then we investigated the influence of the ambiguities of the theoretical parameters on χ_2^2. Since the matter effect causes the difference of the oscillation probabilities between neutrinos and antineutrinos, we have to estimate the fake asymmetry to search for the CP violation effect. However, we will always "overestimate" the fake CP violation due to the ambiguities of the theoretical parameters, and hence we will always estimate the genuine CP-violation effect too small. The matter effect increases as baseline length increases, and we will lose the sensitivity to the asymmetry due to the genuine CP-violation effect in longer baseline such as several thousand km.

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